ANALYSIS OF THE PROCESS OF OPERATING AN AIRCRAFT FROM ASPECT OF FLIGHT SAFETY AND EFFECTIVENESS

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Abstract

This paper presents the method of evaluation of a technical object as immersed in a logistic system with regard to its durability, the operational availability/readiness and the utilisation factor. These factors decide upon the object's effectiveness. For this evaluation probabilistic models from the theory of reliability and of the semi-Markov process were used. For the purpose of this paper as a technical object serves an aircraft.

The process of operating an aircraft consists in using its resources to function accumulated during the process of its manufacture and resulting from a periodic regeneration of these resources in order to maintain its capability of being further operated. For the sake of better understanding our discussion we will refer to the said 'flying craft' as an aircraft (including also other 'means of transport' into this notion). The paper in principle concerns on analysis of safe durability and aircraft availability/readiness analysis. The availability factor of an object in a logistic system expresses the prospective possibility of an effective action. The degree of utilisation of this „potential" for a passenger plane is represented by the occupancy factor. A product ctors of availability and occupancy can describe the overall effectiveness indicator, which characterises the availability and the market demand.

Keywords*: transport, air transport, durability, relibility, semi-Markov process, availability factor*

1. Problem outline

A flying craft operated as a component of a logistic system is characterised in a similar way as every technical object by certain general features as follows:

- it has a determined goal to achieve (a set of goals to be achieved),
- its capability of being operated is limited (a limited operational potential),
- it must be serviced to some extent, it has certain material and power needs, a need of information and other needs,
- its life is limited.

The process of operating an aircraft consists in using its resources to function accumulated during the process of its manufacture and resulting from a periodic regeneration of these resources in order to maintain its capability of being further operated. For the sake of better understanding our discussion we will refer to the said 'flying craft' as an aircraft (including also other 'means of transport' into this notion).

Another feature of the quality of the process of operating a plane is the operational safety and effectiveness. The complexity of problems of safety and effectiveness and the diversity of used definitions require an individual approach to every kind of a system and to set targets, the effectiveness measures of which have to be determined.

The plane safety depends among other things on the intensity of destruction processes of individual elements, functional systems and failure generating systems including the failure rate.

Possible factors, which determine the effectiveness of the process of operating a plane, as considered in a logistic system, are the following:

- the availability / readiness for a flight,
- the utilisation degree during realisation of missions/flights.

2. Analysis of safe durability

For the puspose of the analysis structural elements of a plane, its fuctional systems and technical systmes can be divided into groups according to consequences of their damages/failures. According to this classification we have failures/damages, which can have fatal/catastrophic consequences, and those which within a time horizon under discussion do not entail such consequences. Elements and systems where failures/damages can generate can be divided into controlled and uncontrolled ones. During plane operation uncontrolled devices where failures/breakdowns can occur should be oversized with regard to their reliability structure and they should be „capable" of recording the number of damages and the number of 'fulfiled functions', e.g. the number of load cycles, the operation time, the number of turn-ons etc., moreover they should be 'capable' of determining their techncial condition at any selcted time point. These data gathered usually under laboratory conditions constitute the basis to determine the degree of correlation of a wear with the number or volume of 'fulfilled functions'.

Examples of the analysis of a destruction process averaged in a set of elements or a group set can base on the frequency of damages or on a set of realisation of random variables of technical parameters of a set/group.

An essential parameter determinded during the process of operation with the help of the computer aided system of reliability analysis, which provides information on ageing and wear of elements, is the parameter of a stream of damages for reparable elements $w(t)$. The main information is here a change of a parameter $w(t)$ as a function of time and an answer if it falls within limits of a tolerance field.

The relation between the reliability density function of the failureless operation $f(t)$ and the damage stream parameter $w(t)$ [3] is used to calculate the limit values of this damage stream parameter:

$$w(t) = f(t) + \int_0^t w(t-\tau)\, f(\tau) d\tau \,. \tag{1}$$

An approximate value of the $w(t)$ is given by:

$$w(t) = w(j\Delta t) = w_j = \frac{n_j(\Delta t)}{N\Delta t}, \tag{2}$$

where:

$f(t)$, $f(\tau)$ - the reliability density fuctions of a failureless operation of an object,
$w(t-\tau)$ - the value of the parameter of an object damage stream at the zero time,
$w(t)$ - the above described value at the end time point of an examiantion,
$n_j(\Delta t)$ - number of objects which failed within an j-th operation time interval $[j\Delta t, (j+1)\,\Delta t]$,
N - number of operated objects.

With the aid of the Laplace transformation for the fuctions $f(t)$ and $W(t)$ and some simplififications ($w = \lambda$, where λ - damage intensity in an exponential reliability model) one can obtain a solution for the exponential distribution of damages and adopt the criterion [3]:

$$\lambda(t) = \begin{cases} \lambda_0 & \text{dla} \quad t < b, \\ \lambda_0 + \lambda_1 (t-b) & \text{dla} \quad t \geq b. \end{cases} \tag{3}$$

An essential problem is to find the b point of a growth of the λ parameter.

Knowing the distribution of points b on the time axis of the process of operating elements or groups a rational distribution of operation tasks for the entire object as well as the determianation of periods of safe operation are possible.

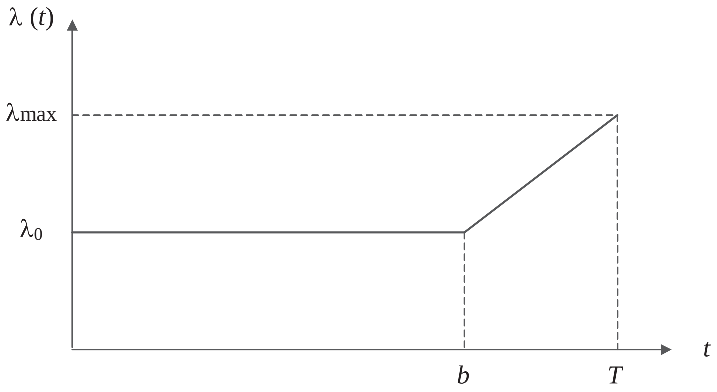

Fig. 1. Example of the stream of damages of ageing elements

3. Aircraft availability/readiness analysis

The probability of dwelling or being maintained at any moment t in a subset of states, which enable an aircraft to be used or for it to accomplish an intended task, is a measure of its technical availability/readiness for action. This technical readiness for action can be also expressed by the expected value of aircraft dwelling in selected states or by other measures, depending on the purpose of the operation process analysis. The technical readiness is a conditional indicator of this readiness of an aircraft to accomplish a task within a time t, provided that other requirements for a possible operation of an aircraft (e.g. provision with fuel, gas, provision with starting devices, etc.) are also met.

Determination of the probability of an aircraft dwell in the availability state for a 4-state operation process model can serve as an example how the analysis method can be useful:
H_1 - the operation status (flight),
H_2 - the stand-by status (maintaining a steady readiness),
H_3 - the status of prophylactic and scheduled repairs,
H_4 - the status of random repairs.

Time periods of aircraft dwelling at individual operational statuses are random variables T_{H_i} of known distributions:

$F_{kH_1}(t) = P\{T_{kH_1} < t\}$ - the cumulative distribution function of the duration time of the k-th flight $k = 1, 2, 3,...,$

$F_{kH_2}(t) = P\{T_{kH_2} < t\}$ - the cumulative distribution function of the time of the k-*th* stand-by $k = 1, 2, 3,...,$

$F(t) = P\{T_{kH_3} < t\}$ - the cumulative distribution function of the time of scheduled services and repairs,

$F(t) = P\{T_{kH_4} < t\}$ - the cumulative distribution function of random repairs.

For an object realising a multiple process of operation the readiness to take on and carry out tasks can be determined by the method of the semi-Markov's analysis of a finished number of phase conditions progressing within a continous time $t \geq 0$.

In order to be able to use the semi-Markovs model reflecting the proces of using an object the conditions as follows must be met:

1. All transitions of an object from an operational status $i \in E$ to an operational status $j \in E$ occur in a sudden form.
2. The dwell time of an objest in an „*i*" state prior to transition into an „*j*" state is a random variable T_{ij} taking on values independent of the calender time.
3. Random variables T_{ij} are inter*in*dependent for every $i, j = 1, 2, 3, ... , r$.

Lets discuss an aircraft operation process composed of a finished set of operational statuses $E = \{1, 2,...,r\}$ represented by a random process $\{Z_t : t \geq 0\}$determined in a following way: $Z_t = i$, if at a time t the process is at the i state. $Z_t = j$, if the process at a time t is in the j state.

The dwell time of the process in the i state prior to transition to the j state is a random variable of a probability distribution function $F_{ij}(t)$, i.e. that probability function determines the said process dwell time in the i state prior to transition to the j state etc. Probabilities of a transition from the state i to the subsequent state j is given by a matrix $r \times r$ $P = [P_{ij}]$, which determines a so-called chain put into the Markov's process. For any $ij \in E$ there exists a determined function $Q_{ij}(t)$, which determines a joint probability of an event consisting in that, that the dwell time of the process in the i state is smaller than t and that from the i state the process will go into the j state.

This function satisfies the following conditions:

$$Q_{ij}(0) = 0;\ i, j \in E,$$
$$\sum_{j \in E} Q_{ij}(\infty) = \sum_{j \in E} P_{ij} = 1,\ i \in E. \tag{4}$$

The set of functions $Q_{ij}(t)$ creates the $Q(t)$ matrix of the process transition distribution $\{Z_t : t \geq 0\}$.

An unconditional probability distribution of the dwell time of the process of operation in the i state is given by the $F_i(t)$ function:

$$F_i(t) = \sum_{j \in E} Q_{ij}(t). \tag{5}$$

An r-dimensional vector of initial probabilities determines the initial status of a process:

$$\boldsymbol{P} = [P_1, P_2, ..., P_r],$$

where:

$$\sum_{i \in E} P_i = 1. \tag{6}$$

A thus determined random process $Z_t : t \geq 0$ is a semi-Markov's one given by a ternary: (r, p, Q).

For a so determined process of operation we can create algorithms to determine many indicators of the aircraft availability with regard to a given subset of tasks or its general availability.

Since in the example under discussion the H_1 and H_2 operational statuses compose a subset H_{12} of technical readiness states the semi-Markov's model of the process of operation $\{Z_t, t \geq 0\}$ can be reduced to three states : H_{12}, H_3, H_4 of the incidence matrix:

$$\mathbf{I} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{7}$$

and the matrix of the process transition distribution:

$$\mathbf{Q}_{ij}(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & 0 \\ Q_{23}(t) & 0 & 0 \end{bmatrix}. \tag{8}$$

The $\boldsymbol{Q}_{ij}(t)$ function depends on the distributions: $G_R(t), F_\tau(t), F_{H_3}(t), F_{H_4}(t)$.

Having determined the $\boldsymbol{Q}_{ij}(t)$ function and the initial distribution $P = \{P_i,\ i = 1,2,3\}$, we can define the indicators of the aircraft technical readiness, assuming, that the H_{12} state is a subset of readiness states.

The transition from the H_{12} state into the H_3 state occurs when the following conditions are satisfied:

- the correct operation time $\tau_R \in T_R$ is greater than the expired overhaul life $\tau_\tau \in T_\tau, \tau_R > \tau_\tau$,
- the operation life till the τ_τ time is not greater than t, $(S_\tau \le t)$.

A joint event $\{(\tau_\tau \le t) \wedge (\tau_R > \tau_\tau)\}$ occurs according to a probability function:

$$Q_{12}(t) = \int_0^t [1 - G_R(y)] dF_\tau(y) . \tag{9}$$

A transition from the H_{12} into the H_4 state occurs when events as follows occur:

- the time of proper operation is not greater than $\tau, (\tau_R \le t)$,
- the overhaul time τ_τ is longer than the time of correct operation $\tau_R \, (\tau_\tau > \tau_R)$.

The joint event $\{(\tau_R < t) \wedge (\tau_\tau > \tau_R)\}$ has a probability function:

$$Q_{13}(t) = \int_0^t [1 - F_\tau(y)] dG_R(y) . \tag{10}$$

A transition from the H_3 state into the H_{12} state occurs when $(T_{H_3} < t)$, therefore:

$$Q_{21}(t) = P\{T_{H_3} < t\}, \tag{11}$$

and from the H_4 into the H_{12} when $(T_{H_4} < t)$, then:

$$Q_{23}(t) = P\{T_{H_4} < t\}. \tag{12}$$

Assuming that the initial state of a process is the H_{12} we obtain the determined initial vector of the process p = [1, 0, 0].

The probability of an aircraft dwell time in the H_{12} technical readiness state over the t time is $1 - F_{12}(t)$. From the formula (13) the following value was determined:

$$P_{12}(t) = 1 - F_{12}(t) = 1 - [Q_{12}(t) + Q_{13}(t)] = 1 - \left\{ \int_0^t [1 - G_r(y)] dF_\tau(y) + \int_0^t [1 - F_\tau(y)] dG_r(y) \right\} . \tag{13}$$

The asymptotic probability of $P_{12}(t)$ of an aircraft dwelling in the technical readiness for $t = \tau$ (flight time) is a product of:

$$P_{12}(t) = P_{12}^{(\infty)} \frac{\int_\tau^\infty [1 - F_{12}(t)] dt}{\int_0^\infty [1 - F_{12}(t)] \, dt} , \tag{14}$$

where $P_{12}^{(\infty)}$ is a limit probability of the aircraft dwell in the H_{12} state, when the process time is very long, $t \to \infty$.

The $P_{12}^{(\infty)}$ value represents the aircraft functional readiness (momentary readiness). The $P_{12}^{(\infty)}$ value is determined from the equation in [3].

If the semi-Markov's process $\{Z_t, t \ge 0\}$ of a finite set E is irreducible and non-periodical and random variables T_{Hj} have finite positive expected values $E[T_{Hj}]$, then limiting transition probabilities exist:

$$p_{ij}^{(\infty)} = \lim_{t \to \infty} p_{ij}(t) \quad i, j \in E , \tag{15}$$

$$p_{ij}^{(\infty)} = p_j^{(\infty)} = \frac{\pi_j E(T_{Hj})}{\sum_{r \in E} \pi_r E(T_{Hr})}. \tag{16}$$

where π_j - denotes a limiting transition probabilities of a Markov's chain inserted into the process. This is the frequency of dwelling of an aircraft in the j-th operational state (a subset of states).

The probability π_j, j = 1, 2,3,... can be calculated by a method given in [3]:

$$\pi_j = \frac{D_j}{\sum_{i \in E} D_i}. \tag{17}$$

where D_i - the principal minor of the matrix P obtained by deleting the i-th column and the j-th line denotes the matrix of transition probabilities of the Markov's chain put into the process,

$$p_{ij} = \lim_{t \to \infty} Q_{ij}(t). \tag{18}$$

Individual values of the p_{ij} in the model under discussion are:

$$p_{12} = \lim_{t \to \infty} Q_{12}(t) = \int_0^\infty [1 - G_{12}(y)] dF_\tau(y), \tag{19}$$

$$p_{13} = Q_{13}(t) = \int_0^\infty [1 - F_\tau(y)] dG_r(y), \tag{20}$$

$$p_{21} = \lim_{t \to \infty} Q_{21}(t) = 1, \tag{21}$$

$$p_{31} = \lim_{t \to \infty} Q_{31}(t) = 1. \tag{22}$$

The matrix P has the following form:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{12} & p_{13} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}. \tag{23}$$

The matrix D is expressed as:

$$\mathbf{D} = \begin{bmatrix} 1 & -p_{12} & -p_{13} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \tag{24}$$

with:

$$\pi_1 = \frac{1}{3 - p_{12} - p_{13}}, \tag{25}$$

$$\pi_2 = \frac{p_{12}}{3 - p_{12} - p_{13}}, \tag{26}$$

$$\pi_3 = \frac{p_{13}}{3 - p_{12} - p_{13}}, \tag{27}$$

$$p_{12}^{(\infty)} = \frac{E[T_{H_{12}}]}{E[T_{H_{12}}] + p_{12} E[T_{H_3}] + p_{13} E[T_{H_4}]}. \tag{28}$$

The expected values of random variables $T_{H_{12}}, T_{H_3}, T_{H_4}$, which are average dwell times of an aircraft in individual operational states can be determined from the following formulas:

$$E[T_{H_{12}}] = \int_0^\infty t[1 - G_R(t)]dF_\tau(t) + \int_0^\infty t[1 - F_\tau(t)]dG_R(t)\,, \tag{29}$$

$$E[T_{H_3}] = \int_0^\infty t dF_{H_3}(t)\,, \tag{30}$$

$$E[T_{H_4}] = \int_0^\infty t dF_{H_4}(t)\,. \tag{31}$$

The expected value of the $T_{H_{12}}$ random variable is also an indicator of the aircraft technical readiness, which characterises an average dwell time in a required subset of states.

A probability of an aircraft dwell time in the technical readiness state determined from the formula (21) is true if a condition that the initial state of the operation process is a state from the set $\{H_1, H_2\}$ is satisfied.

If the operation process initial state differs from the state of readiness, one shall determine the cumulative distribution function of the T_{iA} random variable of the first transition from the initial state $i \in E$, which is not a readiness state, to the A subset – the readiness states set $A \in E$. The cumulative distribution function $T_{iż}$ can be determined from the equation given in [3].

If the A is a subset of states achievable from every state $i \in E \wedge i \notin A$ of the semi-Markov's process the set E of finite states and the $Q_{ij}(t)$ functions are continuous, then there are cumulative distribution functions $F_{iA}(t)$ of the random variable T_{iA}, which satisfy the system of equations:

$$F_{iA}(t) = \sum_{j \in A} Q_{ij}(t) + \sum_{\substack{k \notin \\ k \in E}} F_{kA}(t - x)dQ_{ik}(x)\,. \tag{32}$$

For the discussed model of aircraft operation:

$$F_{3A}(t) = F_{H_3}(t)\,, \tag{33}$$

$$F_{4A}(t) = F_{H_4}(t)\,. \tag{34}$$

The probability $P^{(t)}_{3,1,2}(\tau)$ of an aircraft dwell in a state of technical readiness H_{12} for the time τ after the operation duration time t, when the initial time was the state H_3, can be calculated from the following formula:

$$P^{(t)}_{3,1,2}(\tau) = \int_0^t [1 - F_{12}(t + \tau - x)]dF_{H3}(x)\,, \tag{35}$$

where the value $1 - F_{12}(t + \tau - x)$ is determined from the relation (10).

The $P^{(t)}_{3,1,2}(\tau)$ probability characterises the aircraft technical readiness for action within the time τ after the time t from the beginning of the process starting at the state H_3.

For a process starting at the moment 0 in the state H_4 we have correspondingly:

$$P^{(t)}_{4,1,2}(\tau) = \int_0^t [1 - F_{12}(t + \tau - x)]dF_{H_4}(x)\,. \tag{36}$$

A more exact analysis of the aircraft technical readiness can be carried out using a more complex model.

An actual flying time or the occupancy of available seats for passengers in case of a passenger aircraft can be both a measure of the aircraft utilisation factor. The summary time of the aircraft operation in the subset of H_{12} states until the time t is a S_t random variable of a distribution determined by the relation:

$$P\{S_t < x\} = \sum_{n=1}^{\infty} F_{H_1}^{(n)}(x)\left[F_{H_2}^{(n-1)}(t-x) - F_{H_2}^{n}(t-x)\right]. \tag{37}$$

where the symbol n is an n-multiple convolution of cumulative distribution functions.

The time increment process of using an aircraft as a function of the calendar time shows, for $t \to \infty$, a normal asymptotic distribution.

The expected distribution value is:

$$E[S_t] = \frac{E[T_{H_1}]}{E[T_{H_1}] + E[T_{H_2}]} t = m(t), \tag{38}$$

and the variance is:

$$D^2[S_t] \sim \frac{D^2(T_{H_1})(E[T_{H_2}])^2 + D^2[T_{H_2}](E[T_{H_1}])^2}{(E[T_{H_1}] + E[T_{H_2}])^3} t = [\sigma(t)]^2 . \tag{39}$$

The utilisation factor of an aircraft being in a state of technical readiness is:

$$\rho = \frac{E[T_{H_1}]}{E[T_{H_1}] + E[T_{H_2}]}, \tag{40}$$

where: $E[T_{H_1}]$, $E[T_{H_2}]$ - the expected values of flight times and stand-by times.

The seat occupancy coefficient γ for passenger planes is:

$$\gamma = \frac{\sum_{i=1}^{u} k_i}{u \cdot N}, \tag{41}$$

where:

k_i - the number of occupied passenger seats during an i-th flight (i = 1, 2, ..., u),

N - the number of avaialble seats in a passenger plane.

The overall factor of the operational effect is:

$$E = \rho \cdot \gamma$$

and it takes values from the range [0, 1].

3. Summary

The process of wear and ageing of various elements is to a different extent related to the time or a ‘volume of functioning’ or to the calendar time.

The above-presented method of determining the time of safe operation applies to elements the diagnostic parameters of which are strongly correlated parameters, which determine the readiness state, what can be identified with the existence of a memory of the past.

Time fractions of existence of individual objects and their set and parts between selected events or classes of events saved in a data bank create appropriate set of random variables of the time of a probabilistic model of the proces of operation. Distributions of these random variables can be described by theoretical models, the pararameters of which characterise individual qualitative features of the process of operation and properties of objects.

The measure of the technical availability of an aircraft characterises an object and a logistic system this object is immersed. The probability value expressing the availability translates into a time percentage of an aircraft, over which this aircraft is available for an air traffic management. This value depends on a program determined by a manufacturer and comprising as follows:

- service schedule and diagnostic checks when operating aircrafts,
- service procedures and other operation related activities.

The thus determined availability factor of an object in a logistic system expresses the prospective possibility of an effective action. The degree of utilisation of this „potential” for a passenger plane is represented by the occupancy factor. A product of the above-mentioned factors of availability and occupancy can describe the overall effectiveness indicator, which characterises the availability and the market demand.

Reference

[1] Żurek, J., *Praktyczna eksploatacja techniki lotniczej – Practical Use of Aviation Technlogy*, I Konferencja Praktycznej Eksploatacji Techniki Lotniczej, Kiekrz 1993.

[2] Barziłovic, E., Mezencew, V., Savenkov, M., *Nadieżnost aviacionnych sistem*, Transport, Moskwa 1982.

[3] Jaźwiński, J., *Inżynieria Niezawodności w Eksploatacji Obiektów Technicznych – Reliability Engineering for Operation of Technical Objects,* Poradnik pod red. J. Migdalskiego, Bydgoszcz 1992.

[4] Grabski, F., *Teoria semi-markowskich procesów eksploatacji obiektów technicznych – Theory of Semi-Markov's Processes of Using Technical Objects,* WSMW Gdynia 1985.

[5] Uszakow, J. A., *Sprawocznik nadieżnosti techniczeskich sistem*, Moskwa 1985.

[6] Woropay, M., Żurek, J., *Model gotowości do działania pojazdów interwencyjnych w systemie transportowym – Model of Readiness for Action of Emergency Vehicles in a System of Transportation*, Bydgoszcz-Radom 2003.

[7] Żurek, J., Ziółkowski, J., *Model gotowości do działania pojazdów interwencyjnych – Model of Readiness for Action of Emergency Vehicles*, X Kongres Eksploatacji Urządzeń Technicznych, Stare Jabłonki 2005.